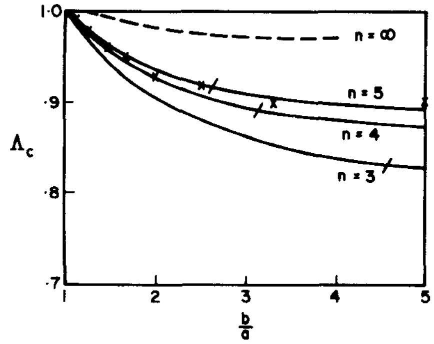

Fig. 5. Normalized cutoff wavelength as function of flat-width ratio. Slashes indicate maximum values of $\frac{b}{a}$ for which theory is expected to be valid. Curve $n = \infty$ is circular case. Crosses are Gruner's results [1] for $n = 4$.

The first requires that the corner regions must be small enough compared with the wavelength that the quasistatic assumptions used to derive the equivalent network constants L and C are justified. Given that we observe that $\Lambda_c \approx 1$, this must always be so.

The other is that the corners are far enough apart so that each can be considered isolated from the others. Essentially this means that a, the length of each of the parallel-plate line segments, should not become much less than comparable with the separation of the plates [5]. The greatest value of s for which the solution is likely to be accurate is $s \approx 1 + 2\tan\pi/n$. For a square coaxial line ($n = 4$), this would give $s \approx 3$ and for an air-filled square coaxial line where $s = 3.5$, $Z_0 \cong 70\Omega$ [6]. It is reasonable to conclude that, for this case, the method is valid for almost all lines likely to be of any practical significance.

In Fig. 5 are shown results computed by this method for $n = 3$, 4, and 5. The slashes indicate the maximum values of s for which the theory would ordinarily be believed to be good. The crosses are sample values from Gruner's numerical solution for the square coaxial line, and are to be compared with our curve for $n = 4$. Agreement is seen to be very good well beyond the range in which the theory is expected to hold. The curve labelled $n = \infty$, the circular coaxial line, is included for comparison, although the present theory is not applicable to it.

III. Generalization of the Method

This technique can be applied to more general cases, such as a line consisting of a rectangle within a rectangle. Even concentricity is not required; all that is needed is that the cross section be made up of sections of parallel-plate line joined by mitred elbows. In this more general case, simplications which result from symmetry are, of course, no longer available. Equation (1) needs to be used to determine resonance and a large number of different matrices will have to be multiplied to determine A, D.

Determination of L and C for each corner could still be undertaken quasistatically on the assumption of isolated corners. In the more general case for L this is easy [7], but for C resorting to some numerical technique such as finite differences would be needed [5]. Valid application of the method continues to rest on having a cross section with small, well-isolated corners.

It may be true though that—unless one enjoys advantages such as the ready availability of a software package for handling finite-difference solutions of Laplace's equation—for these more general cases, if a precise answer is required the cross-sectional resonance technique begins to lose its advantage over a purely numerical solution. On the other hand, if a bound on the answer is all that is required, this method would indicate that a good opening approximation is simply to assume that the cutoff wavelength equates to the mean line circumference.

IV. Conclusions

A theoretical development has been given which allows approximate determination of the cutoff wavelength of the first higher order mode in any transmission line consisting of a pair of coaxial, similar, similarly oriented regular polygons. Comparison for the case of a square coaxial line with results obtained by a purely numerical method indicates that agreement within a few percent is to be expected for all lines having characteristic impedances likely to be of practical interest. Moreover, even without solving the transcendental equation which this approach produces, it is possible to put bounds on the normalized cutoff wavelength of the first higher order mode. If the problem is simply to avoid exciting it, this alone may be enough. It has also been shown that this method is capable of handling more general problems that do not exhibit a high degree of symmetry.

Acknowledgment

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An Explicit Six-Port Calibration Method using Five Standards

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***Abstract*—A six-port reflectometer calibration method using five standards is developed, and gives explicit unambiguous expressions for the calibration constants. The standards are restricted only in that their impedances may neither all have the same magnitude nor all have the same**

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argument. The method permits exact (nonidealized) descriptions of the standards to be used, and the redundancy inherent in the analysis is utilized to reduce the problems associated with measurement noise.

I. Introduction

Various methods for calibrating six-port reflectometers have been suggested in the literature [1]–[6]. Important differences between these methods include the number of calibrating standards required, restrictions on the type of standards, and the amount of computational effort needed to find the calibration constants. Some calibration algorithms [5], [6] assume the use of ideal lossless standards having $|\Gamma|=1$, but this leads to measurement inaccuracies since practical standards are never lossless. When the highest reflectometer accuracy is sought, it is important that the calibration algorithm admit accurate descriptions of the practical (nonideal) standards used.

This paper presents an explicit (noniterative) calibration method requiring five standards. None of the standards is assumed ideal, but it is suggested that one be near a match in order to improve the performance of the calibrated reflectometer near the centre of the Smith chart [7]. The mathematical approach is reminiscent, in part, to that of Li and Bosisio [5], and results in an algorithm which is computationally rapid.

II. Initial Concepts

A six-port reflectometer has power meters connected to the four ports designated $i=1$, 2, 3, and 4, and a termination with reflection coefficient Γ_t' connected to the measuring port. From the theory of six-port junctions [8], the ratio $P_{i,t}$ of power measured at port i to that measured at port 4 can be expressed as

$$P_{i,t}=q_i\left|\frac{1+A_i\Gamma_t'}{1+A_4\Gamma_t'}\right|^2, \qquad i=1,2,3 \tag{1}$$

which has a solution in the form

$$\Gamma_t'=\frac{\sum_{i=1}^{4}F_iP_{i,t}+j\sum_{i=1}^{4}G_iP_{i,t}}{\sum_{i=1}^{4}H_iP_{i,t}} \tag{2}$$

where the F_i, G_i, and H_i are given in terms of the q_i and A_i in the Appendix. The six-port is therefore calibrated once the three real constants q_i and four complex constants A_i have been determined.

The calibration method developed here uses five standard terminations having reflection coefficients $\Gamma_k (k=1,\cdots,5)$ in the desired Z_0 system of measurement. The analysis would be greatly simplified if one of these standards had a reflection coefficient of zero, but this is impossible to realize in practice. However, we can achieve the same analytical simplification by electing to calibrate the reflectometer to measure reflection coefficients Γ' in a system with the impedance of (say) Z_5 of the fifth standard. In such a system, the standards are described by

$$\Gamma_k'=\frac{\Gamma_k-\Gamma_5}{1-\Gamma_k\Gamma_5}, \qquad k=1,\cdots,5 \tag{3}$$

and, in particular, $\Gamma_5'=0$. Because of this normalization, a reflection coefficient Γ_t' calculated from (2) is that for a Z_5 system, but is readily transformed into that for a Z_0 system by

$$\Gamma_t=\frac{\Gamma_t'+\Gamma_5}{1+\Gamma_t'\Gamma_5}. \tag{4}$$

To calibrate the six-port, the five standards are connected successively to the measuring port, and the power ratios $P_{i,k}$ recorded. Since we have normalized to ensure $\Gamma_5'=0$, it follows immediately from (1) that

$$q_i=P_{i,5}, \qquad i=1,2,3 \tag{5}$$

and it only remains to determine the A_i from (1), which has simplified to

$$T_{i,k}=\frac{P_{i,k}}{P_{i,5}}=\left|\frac{1+A_i\Gamma_k'}{1+A_4\Gamma_k'}\right|^2, \qquad i=1,2,3. \tag{6}$$

III. Determination of the A_i

It is convenient at the outset to separate the real and imaginary parts of the A_i and Γ_k', and define

$$a_i+jb_i=A_i$$
$$c_k+js_k=\frac{\Gamma_k'}{|\Gamma_k'|^2}. \tag{7}$$

For each port i, (6) may be expanded as

$$|A_i|^2+2c_ka_i-2s_kb_i=R_{i,k}, \qquad k=1,2,3,4 \tag{8}$$

where

$$R_{i,k}=\frac{T_{i,k}-1}{|\Gamma_k'|^2}+T_{i,k}\left[|A_4|^2+2c_ka_4-2s_kb_4\right]. \tag{9}$$

Eliminating the $|A_i|$, a_i, and b_i from the four expressions in (8) gives

$$f_i|A_4|^2+g_ia_4-h_ib_4+e_i=0 \tag{10}$$

where

$$f_i=\sum_{k=1}^{4}T_{i,k}\gamma_k$$
$$g_i=2\sum_{k=1}^{4}T_{i,k}\gamma_kc_k$$
$$h_i=2\sum_{k=1}^{4}T_{i,k}\gamma_ks_k$$
$$e_i=\sum_{k=1}^{4}\frac{(T_{i,k}-1)\gamma_k}{|\Gamma_k'|^2} \tag{11}$$

and one form of the γ_k is given in the Appendix.

From (10), and a similar expression for port j, the components of A_4 are calculable as

$$a_4=\frac{|A_4|^2\xi_2+\xi_3}{\xi_1}$$
$$b_4=\frac{|A_4|^2\xi_4+\xi_5}{\xi_1} \tag{12}$$

where

$$|A_4|^2=M_{i,j}-\left(M_{i,j}^2-N_{i,j}\right)^{1/2} \tag{13}$$
$$M_{i,j}=\frac{\xi_1^2/2-\xi_2\xi_3-\xi_4\xi_5}{\xi_2^2+\xi_4^2}$$
$$N_{i,j}=\frac{\xi_3^2+\xi_5^2}{\xi_2^2+\xi_4^2}$$

$$\begin{aligned}\xi_1 &= g_i h_j - h_i g_j\\ \xi_2 &= h_i f_j - f_i h_j\\ \xi_3 &= h_i e_j - e_i h_j\\ \xi_4 &= g_i f_j - f_i g_j\\ \xi_5 &= g_i e_j - e_i g_j.\end{aligned} \tag{14}$$

Having computed A_4, the four values $R_{i,k}$, $k = 1, 2, 3$, and 4 for each i may be calculated from (9). Then, any three values l, m, n of k allow the components of the remaining three A_i to be computed from (8) as

$$a_i = \frac{R_{i,l}(s_m - s_n) + R_{i,m}(s_n - s_l) + R_{i,n}(s_l - s_m)}{2[c_l(s_m - s_n) + c_m(s_n - s_l) + c_n(s_l - s_m)]}$$

$$b_i = \frac{R_{i,l}(c_m - c_n) + R_{i,m}(c_n - c_l) + R_{i,n}(c_l - c_m)}{2[c_l(s_m - s_n) + c_m(s_n - s_l) + c_n(s_l - s_m)]}, \quad i = 1,2,3. \tag{15}$$

IV. Discussion and Results

The q_i and A_i are calculable from (5), (12), and (15), and then the F_i, G_i, and H_i from the Appendix. After these calculations, a direct measure of the quality of the calibration is obtained if the values $P_{i,k}$ $(k = 1, \cdots, 5)$ are substituted into (2), and the results compared with the Γ_k'.

An examination of (11) shows that the analysis collapses if all $\gamma_k = 0$. This precludes the use of five standards whose impedances all have the same magnitude, or all have the same argument. Because of their availability, it is convenient (but not necessary) to choose four standards to be offset short circuits, and the fifth Z_5 to be near a match in the desired Z_0 system of measurement. In order to ensure a reasonably even distribution of standards on the Smith chart, it is beneficial to phase the offset short circuits approximately 90° apart.

Four forms of expression can be found for the γ_k by eliminating the $|A_i|$, a_i, and b_i, from (8) in any one of four different ways. Each of these forms will yield all $\gamma_k = 0$ for particular permutations of the four standards Γ_k', $(k = 1, 2, 3$, and 4) if these have equal magnitudes and are symmetrically disposed on the Smith chart. For the form of γ_k given in the Appendix, this potential difficulty is avoided if the two standards having the most positive values of c_k are assigned the indices $k = 1$ and $k = 3$.

The three possible combinations of i and j which lead to (12) give three expressions for a_4, b_4, two of which are redundant. Similarly, there are four expressions available for each a_i, b_i, $(i = 1,2,3)$ from (15), three of which are redundant. In practice, these expressions do not result in identical values. Gross discrepancy between the values is an early indication of gross inconsistency in the calibration. The effect of measurement noise, connector scatter, and imperfect description of the standards causes small discrepancies, and may be reduced by averaging the values.

We now consider the choice of sign in (13). Since it is only meaningful to define the ratio in (1) if the power measured at port 4 is never zero, reflectometers used for the measurement of passive terminations have $0 \leqslant |A_4| < 1$. Since $N_{i,j} \geqslant 0$ and therefore $|M_{i,j}| \geqslant (M_{i,j}^2 - N_{i,j})^{1/2}$, it follows from (13) that $M_{i,j}$ must be positive or zero to satisfy the constraints on $|A_4|$. In the particular circumstance $|\Gamma_k'| = C$ $(k = 1, 2, 3$, and 4) where C is a constant, then $\Sigma\gamma_k = 0$, $e_i = f_i/C$, and $N_{i,j} = 1/C^4 \geqslant 1$. Therefore $M_{i,j} \geqslant 1$, and the negative sign in (13) is necessary. Any

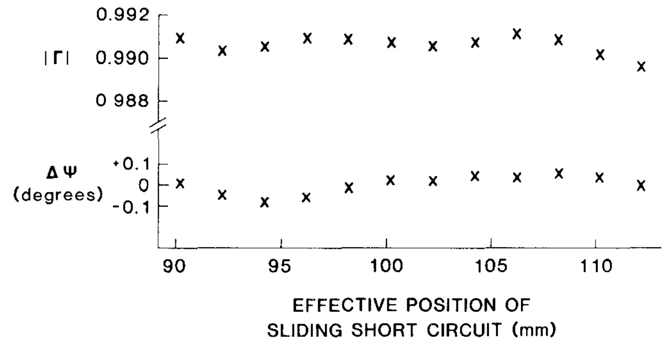

Fig. 1. A demonstration of the capability of a six-port reflectometer calibrated using five standards. The measured magnitude and phase of the reflection coefficient of a precision coaxial sliding short circuit at 7 GHz are shown, with the phase plotted as deviation from nominal.

doubt about the general applicability of this negative sign is readily resolved in practice after reprocessing the calibration data using the opposite sign, and applying the quality of calibration test discussed above.

The form of (1) is invalid if any power meter responds solely to the signal reflected from the measuring port. (If $\Gamma_k' = 0$, then $P_{i,k} = 0$ forcing $q_i = 0$). Such six-port designs are not favored because of the large dynamic range demanded of the power meter [8], but a suitable calibration algorithm paralleling that given here, and based on an alternative expression to (1), is available from the authors.

The calibration algorithm has been implemented on a desk-top computer and applied to the six-port junction of Somlo and Hunter [4], with the power measured either by Schottky diodes calibrated *in situ*, or by a recently developed accurate technique using uncalibrated diodes [9]. For coaxial measurements, four standards are realized using a sliding short circuit whose losses and micrometer error are known from the application of a technique described elsewhere [4]. The fifth standard is a load element sliding in a precision coaxial line (nominal impedance $Z_0 = 50\ \Omega$) with Z_5 calculated from the measured line dimensions and the theoretical properties of the line materials. The effect on $P_{i,5}$ of the residual reflection of the load element is reduced by setting it to three positions along the line, and applying an averaging algorithm [7]. The calibration quality is such that the magnitude of the difference between the known reflection coefficient Γ_k' of each standard, and that computed from (2) using the $P_{i,k}$, is less than 10^{-4}.

Tests with the fifth standard significantly different to Z_0 $(|\Gamma_5| > 0.2)$ revealed increasing sensitivity of the results to the measurement noise.

As a demonstration of the capability of a six-port reflectometer system calibrated using these five standards, the magnitude and phase of the reflection coefficient of a precision coaxial sliding short circuit were measured at 7 GHz. The short circuit was moved in equal increments over a half-wavelength, and the results are shown in Fig. 1. The standard deviation of the magnitude was 0.0004, and the measured phase tracked the nominal phase within 0.1° with a standard deviation of 0.032°, equivalent to a positioning error of less than 2 μm.

V. Conclusions

A method requiring five standards has been presented for the calibration of a six-port reflectometer. Accurate descriptions of practical standards are admitted, and the calibration constants are given by explicit unambiguous formulas which are readily programmable and rapidly evaluated. It is shown how the redundancy inherent in the analysis may be used to reduce the effects of measurement noise, connector scatter, and imperfect descriptions of the calibrating standards.

It is suggested that, in practice, the standards be four short circuits offset by approximately 90°, and a near match. These standards are convenient because of their availability, and beneficial in that their distribution is likely to avoid the accuracy degradation which can occur when measuring in areas of the Smith chart remote from a calibrating standard. Results are presented which demonstrate the viability of the calibration method.

A BASIC listing of the calibration algorithm is available from the authors.

Appendix

Expressions [5] for the constants F_i, G_i, and H_i in terms of the q_i and A_i, and for γ_i are given here in compact programmable form:

$$F_i = \frac{(-1)^i}{2q_i}\left[|A_j|^2(b_k - b_l) + |A_k|^2(b_l - b_j) + |A_l|^2(b_j - b_k)\right]$$

$$G_i = \frac{(-1)^i}{2q_i}\left[|A_j|^2(a_k - a_l) + |A_k|^2(a_l - a_j) + |A_l|^2(a_j - a_k)\right]$$

$$H_i = \frac{(-1)^i}{q_i}\left[|A_j|^2(a_k b_l - a_l b_k) + |A_k|^2(a_l b_j - a_j b_l) + |A_l|^2(a_j b_k - a_k b_j)\right]$$

$$\gamma_i = (c_j - c_k)\left[(s_i - s_j)(c_k - c_l) - (c_i - c_j)(s_k - s_l)\right] + (c_k - c_l)\left[(s_l - s_i)(c_j - c_k) - (c_l - c_i)(s_j - s_k)\right]$$

where

$i = 1, 2, 3$, and 4,

$j = i+1$, $k = i+2$, and $l = i+3$,

$A_i = A_{i+4} = a_i + jb_i$,

$q_4 = 1$, and

$\gamma_i = \gamma_{i+4}$.

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Letters

Comments on "Theory and Measurement of Back Bias Voltage in IMPATT Diodes"

S.C. TIWARI

In the above paper,[1] back bias voltage in IMPATT diodes has been discussed in detail; however, some previous work on this problem has gone unnoticed. Bracket [1] first pointed out that RF-induced negative resistance was responsible for low-frequency instability which was ten times or so higher in GaAs as compared to Si diodes. Using sinusoidal RF voltage, he considered rectification in the avalanche region which showed that the dc operating voltage decreased with increasing RF voltage amplitude. Lee *et al.* [2] first discussed anomalous rectification in dc current when second-order terms in voltage were considered in their analysis. We have also independently found [3], [4] the existence of abnormal rectification in our self-consistent nonlinear avalanche region analysis. It is the purpose of this paper to briefly report relevant results.

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The standard nonlinear integro-differential Read equation is solved self-consistently based on a functional relation between avalanche generated current density $J_{ca}(t)$ and the avalanche region electric field $E_a(t)$ under simplifying assumptions discussed in [3] and [5]. Although the effect of reverse saturation current has also been considered, we write the expressions for $J_s = 0$ given by

$$J_{ca}(t) = J_{ca}(0)\exp(K_i \sin\omega t/\tau_i\omega) \tag{1}$$

$$E_a(t) = b/(\ln(ax_a) - \ln(1 + K_i\cos\omega t))^{1/m} \tag{2}$$

where $J_{ca}(0)$ is $J_{ca}(t)$ at $t = 0$, τ_i is the intrinsic response time, K_i is the injection parameter which determines the RF voltage amplitude, a, b, and m are ionization rate parameters, and x_a is the avalanche region width. The Fourier components of J_{ca} and E_a can be calculated using (1) and (2), and the standard drift region analysis (e.g., [5]) is used to calculate various quantities of interest. The results of calculation for GaAs diodes using ionization rate parameters measured by Salmer *et al.* [6] are presented

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